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Joint Robust Weighted LMMSE Transceiver Design for Dual-Hop AF Multiple-Antenna Relay Systems

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Abstract—In this paper, joint transceiver design for dual-hop amplify-and-forward (AF) MIMO relay systems with Gaussian distributed channel estimation errors in both two hops is investigated. Due to the fact that various linear transceiver designs can be transformed to a weighted linear minimum mean-square-error (LMMSE) transceiver design with specific weighting matrices, weighted mean square error (MSE) is chosen as the performance metric. Precoder matrix at source, forwarding matrix at relay and equalizer matrix at destination are jointly designed with channel estimation errors taken care of by Bayesian philosophy. Several existing algorithms are found to be special cases of the proposed solution. The performance advantage of the proposed robust design is demonstrated by the simulation results.

I. INTRODUCTION

By deploying relays, cooperative communication has great potential to improve system performance [1]. Among various relaying strategies, amplify-and-forward (AF) scheme is attractive for practical implementation due to its low complexity. On other hand, it is well-established that employing multiple antennas is beneficial to improve wireless system performance due to spatial diversity and multiplexing gains. Consequently, in order to obtain the virtues of these two techniques, combination of AF transmission and multi-input multi-output (MIMO) systems attracts more and more researchers' interest.

Linear transceiver design for AF MIMO relay systems has been widely researched in [2]–[6]. Unfortunately, most of the existing works assume channel state information (CSI) is perfectly known. However channel estimation errors are inevitable in practical systems. Robust design, which takes channel estimation errors into account, is therefore desirable in practice. Joint robust design of relay forwarding matrix and destination equalizer under channel estimation errors has been considered in [7]. However in [7], source precoder design, which provides further degree of freedom for improving system performance, is not considered. In this paper, we take a step further to jointly design source precoder matrix, relay forwarding matrix and destination equalizer matrix under channel estimation errors.

There are two main kinds of criteria for transceiver design: capacity maximization and mean-square-error (MSE)

minimization. In fact, both kinds of criteria can be formulated as weighted MSE minimization problems [8]. Therefore, weighted MSE is chosen as the objective function in this paper.

In this paper, the channel estimation errors are assumed to be Gaussian distributed, which is generally true when linear channel estimation algorithms are adopted [9]. The structures of the optimal solutions for the robust joint transceiver design with minimum weighted MSE as the objective are first derived. Based on these optimal structures, the transceiver design is then significantly simplified and iterative water-filling is adopted to solve the problem. Finally, the performance gain of the proposed robust design is verified by simulation.

The following notations are used throughout this paper. Boldface lowercase letters denote vectors, while boldface uppercase letters denote matrices. The notations \mathbf{Z}^T , \mathbf{Z}^* and \mathbf{Z}^H denote the transpose, conjugate and conjugate transpose of the matrix \mathbf{Z} , respectively and $\text{Tr}(\mathbf{Z})$ is the trace of the matrix \mathbf{Z} . The symbol \mathbf{I}_M denotes an $M \times M$ identity matrix, while $\mathbf{0}_{M,N}$ denotes an $M \times N$ all zero matrix. The notation $\mathbf{Z}^{1/2}$ is the Hermitian square root of the positive semi-definite matrix \mathbf{Z} , such that $\mathbf{Z}^{1/2}\mathbf{Z}^{1/2} = \mathbf{Z}$ and $\mathbf{Z}^{1/2}$ is also a Hermitian matrix. The symbol \mathbb{E} denotes statistical expectation.

II. SYSTEM MODEL

A. Transmitted and Received Signals

In this paper, a three node dual-hop amplify-and-forward (AF) cooperative communication system is considered. In the considered system, there is one source with N_S antennas, one relay with M_R receive antennas and N_R transmit antennas, and one destination with M_D antennas. Due to long distance and possibly deep fading, the direct link between the source and destination is not considered in this paper. At the first hop, the source transmits data to the relay. The received signal, \mathbf{x} , at the relay is

$$\mathbf{x} = \mathbf{H}_{sr}\mathbf{P}\mathbf{s} + \mathbf{n}_1 \quad (1)$$

where \mathbf{H}_{sr} is the MIMO channel matrix between the source and the relay, and \mathbf{P} is the precoder matrix at the source. The vector \mathbf{s} is the $N \times 1$ data vector transmitted by the source with the covariance matrix $\mathbf{R}_s = \mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_N$. Furthermore, \mathbf{n}_1 is the additive Gaussian noise vector with correlation matrix $\mathbf{R}_{n_1} = \sigma_1^2 \mathbf{I}_{M_R}$.

At the relay, the received signal \mathbf{x} is multiplied by a forwarding matrix \mathbf{F} . Then the resultant signal is transmitted

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to the destination. The received signal \mathbf{y} at the destination can be written as

$$\mathbf{y} = \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{P}\mathbf{s} + \mathbf{H}_{rd}\mathbf{F}\mathbf{n}_1 + \mathbf{n}_2, \quad (2)$$

where \mathbf{H}_{rd} is the MIMO channel matrix between the relay and the destination, and \mathbf{n}_2 is the additive Gaussian noise vector at the second hop with correlation matrix $\mathbf{R}_{n_2} = \sigma_2^2 \mathbf{I}_{M_D}$. In order to guarantee the transmitted data \mathbf{s} can be recovered at the destination, it is assumed that N_S , M_R , N_R , and M_D are greater than or equal to N [2].

B. Channel Estimation Error Model

Channel state information is usually estimated via using training sequences. In a dual-hop system, there are two channels to be estimated. For implementation simplicity, training sequences are transmitted from both source and destination to the relay. At the relay, the two channels are separately estimated and then based on the channel estimates, transceivers are jointly designed. After that, the designed transceiver matrices are forward to the source and destination, respectively. In general, two hop channels can be written as

$$\mathbf{H}_{sr} = \bar{\mathbf{H}}_{sr} + \underbrace{\sum_{sr}^{1/2} \mathbf{H}_{W, sr} \Psi_{sr}^{1/2}}_{\triangleq \Delta \mathbf{H}_{sr}}, \quad \mathbf{H}_{rd} = \bar{\mathbf{H}}_{rd} + \underbrace{\sum_{rd}^{1/2} \mathbf{H}_{W, rd} \Psi_{rd}^{1/2}}_{\triangleq \Delta \mathbf{H}_{rd}}, \quad (3)$$

where $\bar{\mathbf{H}}_{sr}$ and $\bar{\mathbf{H}}_{rd}$ are the estimated channels, and $\Delta \mathbf{H}_{sr}$ and $\Delta \mathbf{H}_{rd}$ are the corresponding channel estimation errors whose elements are zero mean Gaussian random variables. Furthermore, the $M_R \times N_S$ matrix $\Delta \mathbf{H}_{sr}$ can be decomposed as $\Delta \mathbf{H}_{sr} = \sum_{sr}^{1/2} \mathbf{H}_{W, sr} \Psi_{sr}^{1/2}$, where the elements of the $M_R \times N_S$ matrix $\mathbf{H}_{W, sr}$ are independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and unit variance. When MMSE channel estimator is used, the row and column covariance matrices of $\Delta \mathbf{H}_{sr}$ can be derived as [9]

$$\Sigma_{sr} = \mathbf{I}_{M_R}, \quad \Psi_{sr}^T = (\sigma_{h_{sr}}^{-2} \mathbf{I}_{N_S} + \frac{1}{\sigma_{n_1}^2} \mathbf{D}_1^* \mathbf{D}_1^T)^{-1} \quad (4)$$

where $\sigma_{h_{sr}}^2$ is channel variance in the first hop and \mathbf{D}_1 is the training sequence used in the first hop. Similarly, the row and column covariance matrices of $\Delta \mathbf{H}_{rd}$, can also be derived to be

$$\Sigma_{rd} = (\sigma_{h_{rd}}^{-2} \mathbf{I}_{M_D} + \frac{1}{\sigma_{n_2}^2} \mathbf{D}_2 \mathbf{D}_2^H)^{-1}, \quad \Psi_{rd}^T = \mathbf{I}_{N_R} \quad (5)$$

where $\sigma_{h_{rd}}^2$ denotes the channel variance of the second hop and \mathbf{D}_2 represents the training sequence used in the second hop. Notice that the directions of training sequences transmitted in the two hops are opposite. Therefore, the results given by (4) and (5) are different.

III. PROBLEM FORMULATION

At the destination, a linear equalizer \mathbf{G} is adopted to detect the data vector \mathbf{s} . The mean-square-error (MSE) matrix is $\mathbb{E}\{(\mathbf{G}\mathbf{y} - \mathbf{s})(\mathbf{G}\mathbf{y} - \mathbf{s})^H\}$, where the expectation is taken with respect to random data, channel estimation errors, and noise. In [7], it is shown that

$$\begin{aligned} & \mathbb{E}\{(\mathbf{G}\mathbf{y} - \mathbf{s})(\mathbf{G}\mathbf{y} - \mathbf{s})^H\} \\ &= \mathbf{G}(\bar{\mathbf{H}}_{rd}\mathbf{F}\mathbf{R}_x\mathbf{F}^H\bar{\mathbf{H}}_{rd}^H + \mathbf{K}_2)\mathbf{G}^H + \mathbf{I}_N - (\mathbf{P}^H\bar{\mathbf{H}}_{sr}^H\mathbf{F}^H\bar{\mathbf{H}}_{rd}^H\mathbf{G}^H) \\ & - (\mathbf{G}\bar{\mathbf{H}}_{rd}\mathbf{F}\bar{\mathbf{H}}_{sr}\mathbf{P}), \end{aligned} \quad (6)$$

where matrices \mathbf{R}_x and \mathbf{K}_2 are defined as

$$\begin{aligned} \mathbf{R}_x &= \mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \bar{\mathbf{H}}_{sr}\mathbf{P}\mathbf{P}^H\bar{\mathbf{H}}_{sr}^H + \mathbf{K}_1 \\ \mathbf{K}_1 &= \text{Tr}(\mathbf{P}\mathbf{P}^H\Psi_{sr})\Sigma_{sr} + \mathbf{R}_{n_1} \\ \mathbf{K}_2 &= \text{Tr}(\mathbf{F}\mathbf{R}_x\mathbf{F}^H\Psi_{rd})\Sigma_{rd} + \mathbf{R}_{n_2}. \end{aligned} \quad (7)$$

It is obvious that \mathbf{R}_x is the covariance matrix of the received signal at the relay. Based on the MSE matrix given by (6), a more general performance metric for transceiver design is the weighted MSE [8]:

$$\begin{aligned} \text{MSE}_W(\mathbf{G}, \mathbf{F}, \mathbf{P}) &= \mathbb{E}\{(\mathbf{G}\mathbf{y} - \mathbf{s})^H \mathbf{W}(\mathbf{G}\mathbf{y} - \mathbf{s})\} \\ &= \text{Tr}[\mathbf{W}\mathbb{E}\{(\mathbf{G}\mathbf{y} - \mathbf{s})(\mathbf{G}\mathbf{y} - \mathbf{s})^H\}], \end{aligned} \quad (8)$$

where \mathbf{W} is a $N \times N$ positive semi-definite weighting matrix. Under transmit power constraints at the source and relay, the optimization problem of transceiver design is formulated as

$$\begin{aligned} \text{P1: } & \min_{\mathbf{G}, \mathbf{F}, \mathbf{P}} \text{MSE}_W(\mathbf{G}, \mathbf{F}, \mathbf{P}) \\ \text{s.t. } & \text{Tr}(\mathbf{P}\mathbf{P}^H) \leq P_s, \quad \text{Tr}(\mathbf{F}\mathbf{R}_x\mathbf{F}^H) \leq P_r. \end{aligned} \quad (9)$$

Notice that based on the definition of \mathbf{R}_x in (7), \mathbf{R}_x is a function of \mathbf{P} . In order to simplify the analysis, we define a new variable

$$\tilde{\mathbf{F}} \triangleq \mathbf{F}\mathbf{K}_1^{1/2} \underbrace{(\mathbf{K}_1^{-1/2}\bar{\mathbf{H}}_{sr}\mathbf{P}\mathbf{P}^H\bar{\mathbf{H}}_{sr}^H\mathbf{K}_1^{-1/2} + \mathbf{I}_{M_R})^{1/2}}_{\triangleq \Pi_P}, \quad (10)$$

based on which $\text{Tr}(\mathbf{F}\mathbf{R}_x\mathbf{F}^H) = \text{Tr}(\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H)$ and the two constraints in the optimization problem (9) becomes independent. Furthermore, with (10) the objective function becomes

$$\begin{aligned} & \text{MSE}_W(\mathbf{G}, \tilde{\mathbf{F}}, \mathbf{P}) \\ &= \text{Tr}[\mathbf{W}\mathbf{G}(\bar{\mathbf{H}}_{rd}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H\bar{\mathbf{H}}_{rd}^H + \mathbf{K}_2)\mathbf{G}^H] \\ & - \text{Tr}(\mathbf{W}\mathbf{G}\bar{\mathbf{H}}_{rd}\tilde{\mathbf{F}}\Pi_P^{-1/2}\mathbf{K}_1^{-1/2}\bar{\mathbf{H}}_{sr}\mathbf{P}) \\ & - \text{Tr}(\mathbf{W}\mathbf{P}^H\bar{\mathbf{H}}_{sr}^H\mathbf{K}_1^{-1/2}\Pi_P^{-1/2}\tilde{\mathbf{F}}^H\bar{\mathbf{H}}_{rd}^H\mathbf{G}^H) + \text{Tr}(\mathbf{W}). \end{aligned} \quad (11)$$

As in the optimization problem (9) there is no constraint on \mathbf{G} , the optimal solution for \mathbf{G} can therefore be written as a function of \mathbf{P} and $\tilde{\mathbf{F}}$ [7], and is given by

$$\mathbf{G} = (\bar{\mathbf{H}}_{rd}\tilde{\mathbf{F}}\Pi_P^{-1/2}\mathbf{K}_1^{-1/2}\bar{\mathbf{H}}_{sr}\mathbf{P})^H(\bar{\mathbf{H}}_{rd}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H\bar{\mathbf{H}}_{rd}^H + \mathbf{K}_2)^{-1}. \quad (12)$$

Substituting (12) into the MSE formulation (11), the weighted MSE can be rewritten as

$$\begin{aligned} & \overline{\text{MSE}}_W(\tilde{\mathbf{F}}, \mathbf{P}) \\ &= \text{Tr}(\mathbf{W}) - \text{Tr}[(\bar{\mathbf{H}}_{rd}\tilde{\mathbf{F}}\Pi_P^{-1/2}\mathbf{K}_1^{-1/2}\bar{\mathbf{H}}_{sr}\mathbf{P}\mathbf{W}^{1/2})^H \\ & \times (\bar{\mathbf{H}}_{rd}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H\bar{\mathbf{H}}_{rd}^H + \mathbf{K}_2)^{-1}(\bar{\mathbf{H}}_{rd}\tilde{\mathbf{F}}\Pi_P^{-1/2}\mathbf{K}_1^{-1/2}\bar{\mathbf{H}}_{sr}\mathbf{P}\mathbf{W}^{1/2})]. \end{aligned} \quad (13)$$

Finally, it can be easily proved that the optimal \mathbf{P} and $\tilde{\mathbf{F}}$ must occur on the boundary [7]

$$\text{Tr}(\mathbf{P}\mathbf{P}^H) = P_s, \quad \text{Tr}(\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H) = P_r. \quad (14)$$

Clearly, it means that for the minimum weighted MSE, both relay and source should transmit at the maximum power. As a result, the optimization problem for joint transceiver design is formulated as

$$\begin{aligned} \text{P2: } & \min_{\tilde{\mathbf{F}}, \mathbf{P}} \overline{\text{MSE}}_W(\tilde{\mathbf{F}}, \mathbf{P}) \\ \text{s.t. } & \text{Tr}(\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H) = P_r, \quad \text{Tr}(\mathbf{P}\mathbf{P}^H) = P_s. \end{aligned} \quad (15)$$

In the following, the structures of the optimal solutions are first derived and then the transceiver design problem is simplified and solved in Section VI.

IV. THE STRUCTURE OF OPTIMAL $\tilde{\mathbf{F}}$

With

$$\begin{aligned}\mathbf{A} &\triangleq (\Pi_{\mathbf{P}}^{-1/2} \mathbf{K}_1^{-1/2} \tilde{\mathbf{H}}_{sr} \mathbf{P} \mathbf{W}^{1/2}) \\ \mathbf{M} &\triangleq \tilde{\mathbf{F}}^H \tilde{\mathbf{H}}_{rd}^H (\tilde{\mathbf{H}}_{rd} \tilde{\mathbf{F}} \tilde{\mathbf{F}}^H \tilde{\mathbf{H}}_{rd}^H + \mathbf{K}_2)^{-1} \tilde{\mathbf{H}}_{rd} \tilde{\mathbf{F}},\end{aligned}\quad (16)$$

the weighted MSE can be further reformulated as

$$\overline{\text{MSE}}_{\mathbf{W}}(\tilde{\mathbf{F}}, \mathbf{P}) = \text{Tr}(\mathbf{W}) - \text{Tr}(\mathbf{A}^H \mathbf{M} \mathbf{A}). \quad (17)$$

Using the matrix inversion lemma, $\mathbf{M} = \mathbf{I} - (\tilde{\mathbf{F}}^H \tilde{\mathbf{H}}_{rd}^H \mathbf{K}_2^{-1} \tilde{\mathbf{H}}_{rd} \tilde{\mathbf{F}} + \mathbf{I})^{-1}$, the weighted MSE further becomes

$$\begin{aligned}\overline{\text{MSE}}_{\mathbf{W}}(\tilde{\mathbf{F}}, \mathbf{P}) &= \text{Tr}(\mathbf{W}) - \text{Tr}(\mathbf{A}^H \mathbf{A}) \\ &\quad + \text{Tr}[\mathbf{A}^H (\tilde{\mathbf{F}}^H \tilde{\mathbf{H}}_{rd}^H \mathbf{K}_2^{-1} \tilde{\mathbf{H}}_{rd} \tilde{\mathbf{F}} + \mathbf{I}_{M_R})^{-1} \mathbf{A}].\end{aligned}\quad (18)$$

It should be highlighted that the two constraints in (15) are independent and for any given \mathbf{P} the optimization problem for $\tilde{\mathbf{F}}$ is equivalent to

$$\begin{aligned}\min_{\tilde{\mathbf{F}}} \quad & \text{Tr}[\mathbf{A}^H (\tilde{\mathbf{F}}^H \tilde{\mathbf{H}}_{rd}^H \mathbf{K}_2^{-1} \tilde{\mathbf{H}}_{rd} \tilde{\mathbf{F}} + \mathbf{I}_{M_D})^{-1} \mathbf{A}] \\ \text{s.t.} \quad & \text{Tr}(\tilde{\mathbf{F}} \tilde{\mathbf{F}}^H) = P_r\end{aligned}\quad (19)$$

where the constant parts independent of $\tilde{\mathbf{F}}$ have been neglected. With the fact that $\tilde{\Psi}_{rd} = \mathbf{I}$ from (5), we have

$$\mathbf{K}_2 = \text{Tr}(\tilde{\mathbf{F}} \tilde{\mathbf{F}}^H \Psi_{rd}) \Sigma_{rd} + \sigma_{n_2}^2 \mathbf{I}_{M_D} = P_r \Sigma_{rd} + \sigma_{n_2}^2 \mathbf{I}_{M_D} \quad (20)$$

which is a constant matrix. Furthermore, based on singular value decomposition

$$\mathbf{A} = \mathbf{U}_A \mathbf{\Lambda}_A \mathbf{V}_A^H, \quad \mathbf{K}_2^{-1/2} \tilde{\mathbf{H}}_{rd} = \mathbf{U}_{rd} \mathbf{\Lambda}_{rd} \mathbf{V}_{rd}^H. \quad (21)$$

where the diagonal elements of $\mathbf{\Lambda}_A$ and $\mathbf{\Lambda}_{rd}$ are in decreasing order. Using Majorization theory, the objective function of (19) can be transformed into a Schur-concave function of the diagonal elements of $(\mathbf{U}_A^H \tilde{\mathbf{F}}^H \tilde{\mathbf{H}}_{rd}^H \mathbf{K}_2^{-1} \tilde{\mathbf{H}}_{rd} \tilde{\mathbf{F}} \mathbf{U}_A + \mathbf{I}_{M_R})^{-1}$. Therefore, the optimal solution of (19) has the following structure [7]

$$\tilde{\mathbf{F}} = \mathbf{V}_{rd,N} \mathbf{\Lambda}_{\tilde{\mathbf{F}}} \mathbf{U}_{A,N}^H \quad (22)$$

where $\mathbf{\Lambda}_{\tilde{\mathbf{F}}}$ is a $N \times N$ diagonal matrix such that the diagonal elements of $\mathbf{\Lambda}_{\tilde{\mathbf{F}}} \tilde{\mathbf{\Lambda}}_{rd}^2 \mathbf{\Lambda}_{\tilde{\mathbf{F}}}$ are in decreasing order. The diagonal matrix $\tilde{\mathbf{\Lambda}}_{rd}$ is the $N \times N$ principal submatrix of $\mathbf{\Lambda}_{rd}$. The matrices $\mathbf{V}_{rd,N}$ and $\mathbf{U}_{A,N}$ are the first N columns of \mathbf{V}_{rd} and \mathbf{U}_A , respectively. As \mathbf{A} is a function of \mathbf{P} , the value of $\mathbf{U}_{A,N}$ will be given in the next section, after giving the structure of optimal \mathbf{P} .

V. THE STRUCTURE OF OPTIMAL \mathbf{P}

Substituting the structure of optimal $\tilde{\mathbf{F}}$ (22) into the definition of \mathbf{M} in (16), and together with the fact that the diagonal elements of $\mathbf{\Lambda}_{\tilde{\mathbf{F}}} \tilde{\mathbf{\Lambda}}_{rd}^2 \mathbf{\Lambda}_{\tilde{\mathbf{F}}}$ are in decreasing order, we have

$$\mathbf{M} = \mathbf{U}_{A,N} \underbrace{[\mathbf{I}_N - (\mathbf{\Lambda}_{\tilde{\mathbf{F}}} \tilde{\mathbf{\Lambda}}_{rd}^2 \mathbf{\Lambda}_{\tilde{\mathbf{F}}} + \mathbf{I}_N)^{-1}]}_{\triangleq \tilde{\mathbf{\Lambda}}_M} \mathbf{U}_{A,N}^H \quad (23)$$

where $\tilde{\mathbf{\Lambda}}_M$ is a $N \times N$ diagonal matrix with diagonal elements in decreasing order. It is also straightforward that (23) is the eigen-decomposition of \mathbf{M} . Based on (23), and the singular value decomposition of \mathbf{A} given in (21), after a straightforward substitution we have the following identity

$$\mathbf{A}^H \mathbf{M} \mathbf{A} = \mathbf{V}_A \tilde{\mathbf{\Lambda}}_A \tilde{\mathbf{\Lambda}}_M \tilde{\mathbf{\Lambda}}_A \mathbf{V}_A^H = \mathbf{V}_A \tilde{\mathbf{\Lambda}}_M \mathbf{V}_A^H \mathbf{A}^H \mathbf{A} \quad (24)$$

where $\tilde{\mathbf{\Lambda}}_A$ is the $N \times N$ principal submatrix of $\mathbf{\Lambda}_A$ and is a diagonal matrix with diagonal elements in decreasing order. Using (24), the weighted MSE given by (17) is rewritten as

$$\begin{aligned}\overline{\text{MSE}}_{\mathbf{W}}(\tilde{\mathbf{F}}, \mathbf{P}) &= \text{Tr}(\mathbf{W}) - \text{Tr}(\mathbf{A}^H \mathbf{M} \mathbf{A}) \\ &= \text{Tr}(\mathbf{W}) - \text{Tr}(\mathbf{V}_A \tilde{\mathbf{\Lambda}}_M \mathbf{V}_A^H \mathbf{A}^H \mathbf{A}).\end{aligned}\quad (25)$$

It is shown in Appendix A that for the minimum MSE the following identity holds

$$\mathbf{V}_A = \mathbf{U}_W, \quad (26)$$

where the unitary matrix \mathbf{U}_W is obtained from the eigen-decomposition of $\mathbf{W} = \mathbf{U}_W \mathbf{\Lambda}_W \mathbf{U}_W^H$ in which $\mathbf{\Lambda}_W$ is a diagonal matrix with decreasing diagonal elements. The above equation will also be useful in the following derivation.

From (26) and the definition of \mathbf{A} in (16) and using the matrix inversion lemma again, the weighted MSE (25) can be further rewritten as

$$\begin{aligned}\overline{\text{MSE}}_{\mathbf{W}}(\tilde{\mathbf{F}}, \mathbf{P}) &= \text{Tr}(\mathbf{W}) - \text{Tr}[(\Pi_{\mathbf{P}}^{-1/2} \mathbf{K}_1^{-1/2} \tilde{\mathbf{H}}_{sr} \mathbf{P}) \mathbf{W}^{1/2} \mathbf{U}_W \tilde{\mathbf{\Lambda}}_M \mathbf{U}_W^H \mathbf{W}^{1/2} \\ &\quad \times (\Pi_{\mathbf{P}}^{-1/2} \mathbf{K}_1^{-1/2} \tilde{\mathbf{H}}_{sr} \mathbf{P})^H] \\ &= \text{Tr}(\mathbf{W}) + \text{Tr}[\mathbf{U}_W \mathbf{\Lambda}_W^{1/2} \tilde{\mathbf{\Lambda}}_M \mathbf{\Lambda}_W^{1/2} \mathbf{U}_W^H (\mathbf{P}^H \tilde{\mathbf{H}}_{sr}^H \mathbf{K}_1^{-1} \tilde{\mathbf{H}}_{sr} \mathbf{P} + \mathbf{I})^{-1}] \\ &\quad - \text{Tr}(\mathbf{W}^{1/2} \mathbf{U}_W \tilde{\mathbf{\Lambda}}_M \mathbf{U}_W^H \mathbf{W}^{1/2}).\end{aligned}\quad (27)$$

Notice that based on the definition of \mathbf{M} in (16), $\tilde{\mathbf{\Lambda}}_M$ is a function of $\tilde{\mathbf{F}}$ only and independent of \mathbf{P} . Only the second term in (27) is a function of \mathbf{P} . Then, the optimization problem for \mathbf{P} becomes

$$\begin{aligned}\min_{\mathbf{P}} \quad & \text{Tr}[\mathbf{U}_W \mathbf{\Lambda}_W^{1/2} \tilde{\mathbf{\Lambda}}_M \mathbf{\Lambda}_W^{1/2} \mathbf{U}_W^H (\mathbf{P}^H \tilde{\mathbf{H}}_{sr}^H \mathbf{K}_1^{-1} \tilde{\mathbf{H}}_{sr} \mathbf{P} + \mathbf{I}_N)^{-1}] \\ \text{s.t.} \quad & \text{Tr}(\mathbf{P} \mathbf{P}^H) = P_s.\end{aligned}\quad (28)$$

For source precoder design, the main difference from forward matrix $\tilde{\mathbf{F}}$ design is that \mathbf{K}_1 is not constant. As mentioned previously $\Sigma_{sr} = \mathbf{I}$ (4), then \mathbf{K}_1 equals to

$$\mathbf{K}_1 = [\text{Tr}(\mathbf{P} \mathbf{P}^H \Psi_{sr}) + \sigma_{n_1}^2] \mathbf{I}_{M_R} \triangleq \eta_p \mathbf{I}_{M_R}. \quad (29)$$

With the power constraint $\text{Tr}(\mathbf{P} \mathbf{P}^H) = P_s$, we have

$$\begin{aligned}\eta_p &= \text{Tr}(\mathbf{P} \mathbf{P}^H \Psi_{sr}) + \sigma_{n_1}^2 \\ &= \text{Tr}(\mathbf{P} \mathbf{P}^H \Psi_{sr}) + \sigma_{n_1}^2 \underbrace{\text{Tr}(\mathbf{P} \mathbf{P}^H) / P_s}_{=1} \\ &= \text{Tr}(\mathbf{P} \mathbf{P}^H (P_s \Psi_{sr} + \sigma_{n_1}^2 \mathbf{I}_{N_S})) / P_s.\end{aligned}\quad (30)$$

From (30), the constraint of the optimization problem (28) becomes as

$$\text{Tr}(\mathbf{P} \mathbf{P}^H) = \text{Tr}[\mathbf{P} \mathbf{P}^H (P_s \Psi_{sr} + \sigma_{n_1}^2 \mathbf{I}_{N_S})] / \eta_p = P_s, \quad (31)$$

based on which the optimization problem (28) is equivalent to

$$\begin{aligned}\min_{\mathbf{P}} \quad & \text{Tr}[\mathbf{U}_W \mathbf{\Lambda}_W^{1/2} \tilde{\mathbf{\Lambda}}_M \mathbf{\Lambda}_W^{1/2} \mathbf{U}_W^H (\mathbf{P}^H 1 / \eta_p \tilde{\mathbf{H}}_{sr}^H \tilde{\mathbf{H}}_{sr} \mathbf{P} + \mathbf{I}_N)^{-1}] \\ \text{s.t.} \quad & \text{Tr}[\mathbf{P} \mathbf{P}^H (P_s \Psi_{sr} + \sigma_{n_1}^2 \mathbf{I}_{N_S})] / \eta_p = P_s.\end{aligned}\quad (32)$$

Defining a new variable,

$$\tilde{\mathbf{P}} = 1 / \sqrt{\eta_p} (P_s \Psi_{sr} + \sigma_{n_1}^2 \mathbf{I}_{N_S})^{1/2} \mathbf{P} \quad (33)$$

the optimization problem (32) becomes as (34) as shown at the top of the next page. This formulation is exactly the same

$$\begin{aligned} \min_{\tilde{\mathbf{P}}} \quad & \text{Tr}[\mathbf{U}_W \mathbf{\Lambda}_W^{1/2} \tilde{\mathbf{\Lambda}}_M \mathbf{\Lambda}_W^{1/2} \mathbf{U}_W^H (\tilde{\mathbf{P}}^H (P_s \mathbf{\Psi}_{sr} + \sigma_{n_1}^2 \mathbf{I}_{N_S})^{-1/2} \tilde{\mathbf{H}}_{sr}^H \tilde{\mathbf{H}}_{sr} (P_s \mathbf{\Psi}_{sr} + \sigma_{n_1}^2 \mathbf{I}_{N_S})^{-1/2} \tilde{\mathbf{P}} + \mathbf{I}_N)^{-1}] \\ \text{s.t.} \quad & \text{Tr}(\tilde{\mathbf{P}} \tilde{\mathbf{P}}^H) = P_s \end{aligned} \quad (34)$$

as the optimization problem (19) for $\tilde{\mathbf{F}}$. Following the same argument for $\tilde{\mathbf{F}}$ and defining unitary matrices \mathbf{U}_{sr} and \mathbf{V}_{sr} based on the following singular value decomposition

$$\tilde{\mathbf{H}}_{sr} (P_s \mathbf{\Psi}_{sr} + \sigma_{n_1}^2 \mathbf{I}_{N_S})^{-1/2} = \mathbf{U}_{sr} \mathbf{\Lambda}_{sr} \mathbf{V}_{sr}^H, \quad (35)$$

with the diagonal elements of the diagonal matrix $\mathbf{\Lambda}_{sr}$ in decreasing order, the optimal $\tilde{\mathbf{P}}$ has the following structure

$$\tilde{\mathbf{P}} = \mathbf{V}_{sr,N} \mathbf{\Lambda}_{\tilde{\mathbf{P}}} \mathbf{U}_W^H \quad (36)$$

where $\mathbf{\Lambda}_{\tilde{\mathbf{P}}}$ is a $N \times N$ diagonal matrix such that the diagonal elements of $\mathbf{\Lambda}_{\tilde{\mathbf{P}}} \tilde{\mathbf{\Lambda}}_{sr}^2 \mathbf{\Lambda}_{\tilde{\mathbf{P}}}$ are in decreasing order. The diagonal matrix $\tilde{\mathbf{\Lambda}}_{sr}$ is the $N \times N$ principal submatrix of $\mathbf{\Lambda}_{sr}$. Furthermore, based on the definition of $\tilde{\mathbf{P}}$ given by (33), the optimal \mathbf{P} has the following structure

$$\mathbf{P} = \sqrt{\eta_p} (P_s \mathbf{\Psi}_{sr} + \sigma_{n_1}^2 \mathbf{I}_{N_S})^{-1/2} \mathbf{V}_{sr,N} \mathbf{\Lambda}_{\tilde{\mathbf{P}}} \mathbf{U}_W^H. \quad (37)$$

Substituting (37) into the definition of \mathbf{A} in (16), we have $\mathbf{U}_{\mathbf{A},N} = \mathbf{U}_{sr,N}$. Therefore, the optimal $\tilde{\mathbf{F}}$ has the following structure

$$\tilde{\mathbf{F}} = \mathbf{V}_{rd,N} \mathbf{\Lambda}_{\tilde{\mathbf{F}}} \mathbf{U}_{sr,N}^H. \quad (38)$$

Remark: Given (37) and (38), the remaining problem is how to determine two diagonal matrices $\mathbf{\Lambda}_{\tilde{\mathbf{F}}}$ and $\mathbf{\Lambda}_{\tilde{\mathbf{P}}}$.

Notice that after $\mathbf{\Lambda}_{\tilde{\mathbf{P}}}$ is computed, the remaining unknown parameter in (37) is only η_p . In order to solve η_p , substitute the formulation of \mathbf{P} (37) into the definition of η_p in (29), and then we get

$$\begin{aligned} \eta_p &= \text{Tr}(\mathbf{P} \mathbf{P}^H \mathbf{\Psi}_{sr}) + \sigma_{n_1}^2 \\ &= \eta_p \text{Tr}[\mathbf{V}_{sr,N}^H (P_s \mathbf{\Psi}_{sr} + \sigma_{n_1}^2 \mathbf{I})^{-1/2} \mathbf{\Psi}_{sr} (P_s \mathbf{\Psi}_{sr} + \sigma_{n_1}^2 \mathbf{I})^{-1/2} \\ &\quad \times \mathbf{V}_{sr,N} \mathbf{\Lambda}_{\tilde{\mathbf{P}}}^2] + \sigma_{n_1}^2. \end{aligned} \quad (39)$$

This is a simple linear function of η_p , and η_p can be easily solved to be

$$\begin{aligned} \eta_p &= \sigma_{n_1}^2 / \{1 - \text{Tr}[\mathbf{V}_{sr,N}^H (P_s \mathbf{\Psi}_{sr} + \sigma_{n_1}^2 \mathbf{I})^{-1/2} \mathbf{\Psi}_{sr} \\ &\quad \times (P_s \mathbf{\Psi}_{sr} + \sigma_{n_1}^2 \mathbf{I})^{-1/2} \mathbf{V}_{sr,N} \mathbf{\Lambda}_{\tilde{\mathbf{P}}}^2]\}. \end{aligned} \quad (40)$$

VI. PROPOSED SOLUTION FOR $\mathbf{\Lambda}_{\tilde{\mathbf{F}}}$ AND $\mathbf{\Lambda}_{\tilde{\mathbf{P}}}$

In this section, the optimal $\mathbf{\Lambda}_{\tilde{\mathbf{F}}}$ and $\mathbf{\Lambda}_{\tilde{\mathbf{P}}}$ will be derived. Notice that $\tilde{\mathbf{\Lambda}}_{sr}$ and $\tilde{\mathbf{\Lambda}}_{rd}$ are the $N \times N$ principal submatrices of $\mathbf{\Lambda}_{sr}$ and $\mathbf{\Lambda}_{rd}$, respectively. Based on (31), substituting the optimal structures (37) and (38) into the original optimization problem (15), and denoting

$$\begin{aligned} \tilde{\mathbf{\Lambda}}_{sr} &= \text{diag}\{\lambda_{sr,i}\} \quad \tilde{\mathbf{\Lambda}}_{rd} = \text{diag}\{\lambda_{rd,i}\} \quad \mathbf{\Lambda}_{\tilde{\mathbf{F}}} = \text{diag}\{f_i\} \\ \mathbf{\Lambda}_{\tilde{\mathbf{P}}} &= \text{diag}\{p_i\} \quad \mathbf{\Lambda}_W = \text{diag}\{w_i\} \end{aligned} \quad (41)$$

the optimization problem (15) becomes

$$\begin{aligned} \min_{f_i, p_i} \quad & \sum_{i=1}^N \frac{w_i (f_i^2 \lambda_{rd,i}^2 + p_i^2 \lambda_{sr,i}^2 + 1)}{(p_i^2 \lambda_{sr,i}^2 + 1) (f_i^2 \lambda_{rd,i}^2 + 1)} \\ \text{s.t.} \quad & \sum_{i=1}^N f_i^2 = P_r \quad \sum_{i=1}^N p_i^2 = P_s, \end{aligned} \quad (42)$$

where $\tilde{\mathbf{\Lambda}}_a = \text{diag}\{\lambda_{a,i}\}$ means that the i^{th} diagonal element of the diagonal matrix $\tilde{\mathbf{\Lambda}}_a$ is denoted as $\lambda_{a,i}$. This optimization is non-convex, and thus generally speaking it is difficult to solve. However, notice that when p_i 's are fixed, f_i 's can be computed as

$$f_i = \left[\left(\sqrt{\frac{w_i}{\mu_f \lambda_{rd,i}^2}} \sqrt{\frac{p_i^2 \lambda_{sr,i}^2}{1 + p_i^2 \lambda_{sr,i}^2}} - \frac{1}{\lambda_{rd,i}^2} \right)^+ \right]^{1/2}, \quad (43)$$

where μ_f is the Lagrange multiplier which makes $\sum f_i^2 = P_r$. On the other hand, when f_i 's are fixed, p_i 's can be computed as

$$p_i = \left[\left(\sqrt{\frac{w_i}{\mu_p \lambda_{sr,i}^2}} \sqrt{\frac{f_i^2 \lambda_{rd,i}^2}{1 + f_i^2 \lambda_{rd,i}^2}} - \frac{1}{\lambda_{sr,i}^2} \right)^+ \right]^{1/2}, \quad (44)$$

where μ_p is the Lagrange multiplier which makes $\sum p_i^2 = P_s$ hold. Notice that this iterative water-filling algorithm is guaranteed to converge, as discussed in [10].

Special cases: Several existing algorithms can be considered as special cases of our proposed solution.

- When CSI is perfectly known, $\mathbf{W} = \mathbf{I}$ and $\mathbf{P} = \mathbf{I}$, the proposed solution for \mathbf{F} reduces to that in [2].
- When CSI is perfectly known and $\mathbf{W} = \mathbf{I}$, the proposed solution for \mathbf{P} and \mathbf{F} reduces to that given in [5].
- When the second hop channel is an identity matrix and noiseless, the proposed solution for source precoder design reduces to that given in [11].

VII. SIMULATION RESULTS AND DISCUSSIONS

In this section, simulation results are presented to demonstrate the performance of the proposed algorithm. For the purpose of comparison, the algorithm based on the estimated channel only (without taking the channel errors into account) [5] and the robust algorithm without source precoder design in [7] are also simulated. In the following, we consider an AF MIMO relay system where the source, relay and destination are equipped with the same number of antennas, i.e., $N_S = M_R = N_R = M_D = 4$. The elements of channel matrices \mathbf{H}_{sr} and \mathbf{H}_{rd} are randomly generated as i.i.d. Gaussian distributed random variables.

The widely used exponential correlation matrix $\mathbf{R}_\alpha = \{\alpha^{|i-j|}\}_{i,j}$ is used to model the correlation matrix of \mathbf{D} , i.e., $\mathbf{D} \mathbf{D}^H \propto \mathbf{R}_\alpha$. Then $\mathbf{\Psi}_{sr} = \mathbf{\Sigma}_{rd} = (\mathbf{I}_4 + \text{SNR}_{\text{EST}} \mathbf{R}_\alpha)^{-1}$ where SNR_{EST} is the signal-to-noise ratio (SNR) in channel estimation process. In the simulation, the weighting matrix is selected as $\mathbf{W} = \text{diag}\{[0.3 \ 0.3 \ 0.2 \ 0.2]\}$. The noise covariance matrices are $\mathbf{R}_{n_1} = \sigma_1^2 \mathbf{I}_4$ and $\mathbf{R}_{n_2} = \sigma_2^2 \mathbf{I}_4$. In data transmission stage the SNRs at relay and destination are defined as P_s/σ_1^2 and P_r/σ_2^2 , respectively.

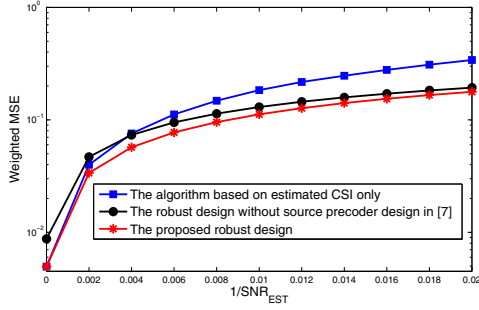


Fig. 1. Weighted MSEs of the detected data for different algorithms, when $\alpha = 0.3$ and $P_s/\sigma_1^2 = P_r/\sigma_2^2 = 30\text{dB}$.

For the source node, four independent data streams are transmitted and in each data stream, $N_{Data} = 10000$ independent QPSK symbols are transmitted. Each point in the figure is an average of 10000 independent channel realizations. Fig. 1 shows the weighted MSEs at the destination for different algorithms when $\alpha = 0.3$ and $P_s/\sigma_1^2 = P_r/\sigma_2^2 = 30\text{dB}$. It can be seen that the performance of the proposed algorithm is always better than that based on the estimated CSI only. It can also be observed that the proposed robust algorithm with source precoder design performs better than that without source precoder design in [7], illustrating the importance of joint transceiver design involving source precoder.

VIII. CONCLUSIONS

Robust LMMSE transceiver design under Gaussian channel uncertainties for dual-hop AF MIMO relay systems was investigated. Exploiting channel estimation error statistics and using a general weighted MSE performance metric, the precoder matrix at the source, forwarding matrix at the relay and equalizer matrix at the destination were jointly optimized. It was found that several existing solutions are special cases of our proposed solution. The performance advantage of the proposed algorithm was demonstrated by the computer simulations.

APPENDIX A

Based on (24) and the fact that $\tilde{\Lambda}_A$ and $\tilde{\Lambda}_M$ are diagonal matrices with diagonal elements in decreasing order, the following identity holds

$$\mathbf{A}^H \mathbf{M} \mathbf{A} = \mathbf{V}_A \mathbf{\Lambda}_1 \mathbf{V}_A^H, \quad (45)$$

where $\mathbf{\Lambda}_1 = \tilde{\Lambda}_A \tilde{\Lambda}_M \tilde{\Lambda}_A$ is a diagonal matrix with diagonal elements in decreasing order. Notice that (45) in fact is an eigen-decomposition of $\mathbf{A}^H \mathbf{M} \mathbf{A}$ with eigenvalues in decreasing order and \mathbf{V}_A is the corresponding unitary matrix.

Defining

$$\mathbf{N} \triangleq (\tilde{\mathbf{H}}_{rd} \tilde{\mathbf{F}} \Pi_P^{-1/2} \mathbf{K}_1^{-1/2} \tilde{\mathbf{H}}_{sr} \mathbf{P})^H (\tilde{\mathbf{H}}_{rd} \tilde{\mathbf{F}} \tilde{\mathbf{F}}^H \tilde{\mathbf{H}}_{rd}^H + \mathbf{K}_2)^{-1} \times (\tilde{\mathbf{H}}_{rd} \tilde{\mathbf{F}} \Pi_P^{-1/2} \mathbf{K}_1^{-1/2} \tilde{\mathbf{H}}_{sr} \mathbf{P}), \quad (46)$$

and together with the definitions of \mathbf{A} and \mathbf{M} in (16) the following equation holds

$$\mathbf{A}^H \mathbf{M} \mathbf{A} = \mathbf{W}^{1/2} \mathbf{N} \mathbf{W}^{1/2}, \quad (47)$$

based on which, the weighted MSE (18) can be rewritten as

$$\begin{aligned} \text{MSE}_W(\tilde{\mathbf{F}}, \mathbf{P}) &= \text{Tr}(\mathbf{W}) - \text{Tr}(\mathbf{W}\mathbf{N}) \\ &\geq \text{Tr}(\mathbf{W}) - \sum_i \lambda_i(\mathbf{W}) \lambda_i(\mathbf{N}), \end{aligned} \quad (48)$$

where $\lambda_i(\mathbf{Z})$ denotes the i^{th} largest eigenvalue of \mathbf{Z} . Using Neumann inequality [12], for the minimum weighted MSE \mathbf{N} and \mathbf{W} have the same eigen-vectors. In other words, given the eigen-decomposition of \mathbf{W} as

$$\mathbf{W} = \mathbf{U}_W \mathbf{\Lambda}_W \mathbf{U}_W^H \quad (49)$$

where the diagonal elements of the diagonal matrix $\mathbf{\Lambda}_W$ are in decreasing order, for the minimum MSE, \mathbf{N} could be eigen-decomposed as

$$\mathbf{N} = \mathbf{U}_W \mathbf{\Lambda}_N \mathbf{U}_W^H. \quad (50)$$

where $\mathbf{\Lambda}_N$ is a diagonal matrix whose diagonal elements are in decreasing order. Based on (49) and (50), for the minimum MSE it holds that

$$\mathbf{A}^H \mathbf{M} \mathbf{A} = \mathbf{W}^{1/2} \mathbf{N} \mathbf{W}^{1/2} = \mathbf{U}_W \underbrace{\mathbf{\Lambda}_W \mathbf{\Lambda}_N \mathbf{U}_W^H}_{=\mathbf{\Lambda}_2}. \quad (51)$$

As the diagonal elements of the diagonal matrices $\mathbf{\Lambda}_W$ and $\mathbf{\Lambda}_N$ are positive and both in decreasing order, the diagonal elements of $\mathbf{\Lambda}_2$ are also in decreasing order. Clearly, the second equation of (51) also denotes an eigen-decomposition of $\mathbf{A}^H \mathbf{M} \mathbf{A}$. Comparing (45) and (51), it can be concluded that for the minimum MSE there exists an eigen-decomposition of $\mathbf{A}^H \mathbf{M} \mathbf{A}$ with eigenvalues in decreasing order such that

$$\mathbf{V}_A = \mathbf{U}_W. \quad (52)$$

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